

## Computer Graphics Ph.D. Qualifying Exam, March 2008

1. (10%) If you draw on a black-and-white display using Bresenham's algorithm, does a  $45^\circ$  line look brighter, the same, or dimmer than one drawn horizontally? Why? (Assume that the display has square pixels that do not overlap and no anti-aliasing.)
2. (15%)  $\mathbf{A}$  and  $\mathbf{B}$  are  $3 \times 3$  rotation matrices. Let  $\mathbf{C}$  be a matrix created by a convex linear combination of them ( $\mathbf{C} = \alpha\mathbf{A} + (1 - \alpha)\mathbf{B}$ ). Under what circumstances will  $\mathbf{C}$  be a rotation matrix?
3. (15%) A point  $\mathbf{p}$  on a surface has an associated unit normal  $\mathbf{n}$ , and tangent vectors  $\mathbf{t}_1$  and  $\mathbf{t}_2$ . If the surface is transformed by the matrix  $\mathbf{M}$  and the transformed point of  $\mathbf{p}$  is  $\mathbf{M}\mathbf{p}$ , what is the transformed normal vector? (Hint: by definition, the normal vector is orthogonal to tangent vectors. The transformed normal should maintain this property.)
4. (25%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[ I_{l_i} \left( k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s} \right) \min \left( 1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

5. (10%) A mipmap is an data structure for texture map anti-aliasing. Explain how it works and estimate its memory consumption compared to the initial image alone.
6. (10%) Describe the rendering equation proposed by Kajiya in his classic SIGGRAPH 1986 paper.
7. (15%) Radiosity and ray tracing are two classic approaches for global illumination. Explain their basic ideas and their strengthes and weaknesses.