博士班基本學科考試：數位影像處理

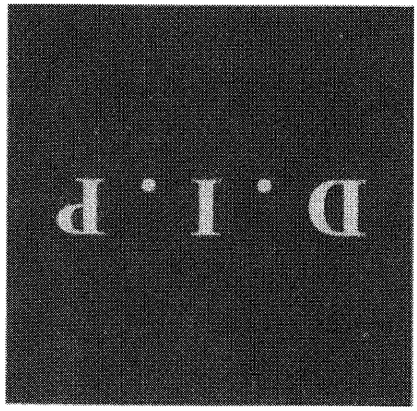
2006 年 3 月 9 日

1. (20%)
   (a) The following table gives the number of pixels at each of the gray levels 0-15 in an image with those gray values only. Draw the histogram corresponding to these gray levels, and then perform a histogram equalization and draw the resulting histogram.
   (b) Write down an algorithm for histogram equalization.
   (c) Is the histogram equalization operation idempotent? Why?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	40	80	45	110	70	0	0	0	0	0	0	0	0	15

2. (20%) Consider the morphological operations for binary images. Prove the duality between the opening and the closing, i.e., $(A \bullet B)^c = (A^c \circ \hat{B})$. (Notice: If you want to use the duality of dilation and erosion, you will have to write down its proof.)

3. (20%) Consider the image shown below. The image on the right was obtained by (a) multiplying the image on the left by $(-1)^{x+y}$; (b) computing the DFT; (c) taking the complex conjugate of the transform; (d) computing the inverse DFT; (e) multiplying the real part of the result by $(-1)^{x+y}$. Explain (mathematically) why the image on the right appears as it does.



4. (20%)
   (a) What is the inverse filter? What is the Wiener filter? Write down their mathematical expression.
   (b) Is the Wiener filter "optimal" in some sense? If yes, in what sense?
   (c) When does the Wiener filter reduce to the inverse filter?

5. (20%)
   A pseudo-median filter has been proposed to overcome some of the speed disadvantages of the median filter. For example, given a five-element sequence $\{a, b, c, d, e\}$, its pseudo-median is defined as

$$\text{psmed}(a, b, c, d, e) = \frac{1}{2}\max\{\min(a, b, c), \min(b, c, d), \min(c, d, e)\} + \frac{1}{2}\min\{\max(a, b, c), \max(b, c, d), \max(c, d, e)\}$$

   So for a sequence of length 5, we take the maxima and minima of all subsequences of length 3. In general, for an odd-length sequence L of length $2n+1$, we take the maxima and minima of all subsequences of length $n+1$.

   (a) Why can the pseudo-median be a good approximation of the median? Notice that the pseudo-median will exhibit a more "center-weighted" response than the median.
   (b) Can you apply the pseudo-median to 3×3 neighborhoods of an image? How?