

資訊網路與多媒體研究所

博士班資格考試試題

科目：資訊理論與編碼技巧

2011/9/30

1. Consider the following method for generating a code for a random variable X which takes on m values $\{1, 2, \dots, m\}$ with probabilities p_1, p_2, \dots, p_m . Assume that the probabilities are ordered so that $p_1 \geq p_2 \geq \dots \geq p_m$. Define

$$F_i = \sum_{k=1}^{i-1} p_k,$$

the sum of the probabilities of all symbols less than i . Then the codeword for i is

the number $F_i \in [0, 1]$ rounded off to l_i bits, where $l_i = \left\lceil \log \frac{1}{p_i} \right\rceil$.

- (1) Show that the code constructed by this process is prefix-free and the average length satisfies

$$H(X) \leq L \leq H(X) + 1.$$

- (2) Construct the code for the probability distribution (0.5, 0.25, 0.125, 0.125).

2. Find the Ternary Huffman codes of the follow discrete sources

(a) $\{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.15), (5, 0.15)\}$

(b) $\{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.1), (5, 0.1), (6, 0.1)\}$

where (x, p) stands for the source symbol and its associated probability.

3. Show that the expected length under $p(x)$ of the code assignment

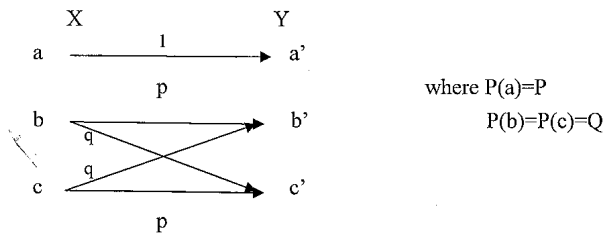
$$l(x) = \left\lceil \log \frac{1}{q(x)} \right\rceil \text{ satisfies}$$

$$H(p) + D(p \parallel q) \leq Epl(x) < H(p) + D(p \parallel q) + 1$$

where $\lceil x \rceil$ is the smallest integer larger than or equal to x .

4. Please draw the basic structure of a motion-compensated video coding scheme. (including both Encoder and Decoder)

5. Find the Channel Capacity of the following channel.



6. Please find the differential entropy of a continuous Gaussian random variable X ,

where $X \sim \varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$