

資訊網路與多媒體研究所

資格考科目：資訊理論與編碼技巧

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1. Find the Ternary Huffman codes of the follow discrete sources

(a)  $\{(1,0.25), (2,0.25), (3,0.2), (4,0.15), (5,0.15)\}$

(b)  $\{(1,0.25), (2,0.25), (3,0.2), (4,0.1), (5,0.1), (6,0.1)\}$

where  $(x, p)$  stands for the source symbol and its associated probability.

2. Show that the expected length  $L$  of any instantaneous  $D$ -ary code for a random variable  $X$  is greater than or equal to the entropy  $H_D(X)$ ; that is

$$L \geq H_D(X)$$

with equality if and only if  $D^{-li} = P_i$ ,

where  $li$  and  $P_i$  are the length and probability of symbol  $x_i$  drawn from  $X$ .

3. Show that the expected length under  $p(x)$  of the code assignment

$$l(x) = \left\lceil \log \frac{1}{q(x)} \right\rceil \text{ satisfies}$$

$$H(p) + D(p // q) \leq Epl(x) < H(p) + D(p // q) + 1$$

where  $\lceil x \rceil$  is the smallest integer larger than or equal to  $X$ .

4. Prove that if  $f$  is a convex function and  $X$  is a random variable

$$Ef(x) \geq f(EX)$$

where  $E$  stands for expectation.