Computer Graphics Ph.D. Qualifying Exam, March 2009

1. (15%) $\mathbf{A}$ and $\mathbf{B}$ are 3×3 rotation matrices. Let $\mathbf{C}$ be a matrix created by a convex linear combination of them ($\mathbf{C} = \alpha\mathbf{A} + (1 - \alpha)\mathbf{B}$). Under what circumstances will $\mathbf{C}$ be a rotation matrix?

2. (15%) Consider a unit square. Assume that the texture coordinates are (u_{00}, v_{00}), (u_{10}, v_{10}), (u_{01}, v_{01}), (u_{11}, v_{11}) for its four vertices $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$. Given a point (x, y) within the square, what is its texture coordinate using bilinear interpolation?

3. (10%) A mipmap is an data structure for texture map anti-aliasing. Explain how it works and estimate its memory consumption compared to the initial texture image alone.

4. (20%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[I_{l_i} \left(k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s} \right) \min \left(1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

   Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

5. (15%) Given a triangle T with vertices V_1, V_2, V_3 and a ray R with origin O and direction d, (a) what is the plane equation where T lies? (5%), (b) what is the intersection of R and that plane? (5%) and (c) how to check whether R and T intersect? (5%)

6. (25%) (a) Describe the rendering equation proposed by Kajiya in his classic SIGGRAPH 1986 paper. (10%) (b) Explain how to derive Whitted's model from the rendering equation. (10%) (c) Could you suggest a way to find the solution to the rendering equation without making assumptions. You can ignore the efficiency issue. (5%)