資訊網路與多媒體研究所

資格考試科目：資訊理論與編碼技巧

Information Theory and Coding Technique

2017/9/28

1. (6%) 試舉出一實例說明 Shannon Entropy 不合應用所需的情況。

2. (16%) A function $\rho(x, y)$ is a metric if for all *x, y,*
   - $\rho(x, y) \geq 0$.
   - $\rho(x, y) = \rho(y, x)$.
   - $\rho(x, y) = 0$ if and only if $x = y$.
   - $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$.

   (1) Show that $\rho(X,Y) = H(X \mid Y) + H(Y \mid X)$ satisfies the first, second, and fourth properties above. If we say that $X = Y$ if there is one-to-one function mapping X to Y, then the third property is also satisfied, and $\rho(X,Y)$ is a metric.

   (2) Verify that $\rho(X,Y)$ can also be expressed as

$$\begin{aligned}\rho(X,Y) &= H(X) + H(Y) - 2I(X;Y) \\ &= H(X,Y) - I(X;Y) \\ &= 2H(X,Y) - H(X) - H(Y).\end{aligned}$$

3. (15%) For the source $X=\begin{Bmatrix} x_1, & x_2, & x_3, & x_4, & x_5, & x_6 \\ 0.25, & 0.25, & 0.2, & 0.1, & 0.1, & 0.1 \end{Bmatrix}$

   Please construct an optimal Tenary Huffman code for X and find the corresponding average codeword length. (15%)

4. (20%) Try your best to derive all the relationships between Type-II DCT and discrete Fourier transform (DFT), where an N-point DFT is defined as

   F(x) $\triangleq \bar{X} = \sum_{n=0}^{N-1} x(n)\, W_N^{nK}$ and $W_N^K = \exp\left(-j\frac{2\pi}{N}\right)$ K

5. (9%) Please briefly explain why and under what conditions DPCM will provide compression gain?

6. (9%) Please briefly explain why and under what conditions Transform coding will provide compression gain?

7. (10%) What is 3-step fast Search Algorithm?
   Please describe the pros and cons of 3-step fast Search Algorithm?

8. (15%) Assume *f(x)* is a convex function and $\overline{X}$ is a set of random variables.
   Prove that $\sum_{i=1}^{k} p_i f(x_i) \geq f\left(\sum_{i=1}^{k} p_i x_i\right)$
   Where $x_i \in \overline{X}$ and p_i is the corresponding probability of x_i.