

Computer Graphics Ph.D. Qualifying Exam, September 2017

1. (20%) Estimate the 2D affine transformation matrix $\mathbf{T} = \begin{bmatrix} \mathbf{F} & v \\ 0^T & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3}$ which has the following mappings on three homogeneous points.

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{T}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{T}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \xrightarrow{\mathbf{T}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

2. (20%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[I_i (k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s}) \min \left(1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (10%) (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences. (10%)

3. (20%) (a) Given a ray whose origin is $\{0, 0, 0\}$ and direction is $\mathbf{D} = \{D_x, D_y, D_z\}$, express the coordinate of a point \mathbf{p} on the ray using its distance t to the ray origin. (5%) (b) Given a sphere with the center $\mathbf{C} = \{C_x, C_y, C_z\}$ and radius r , the intersection of the sphere and the above ray can be found by solving a quadratic equation of t , $At^2 + Bt + C = 0$. What are A , B and C ? (15%)
4. (20%) Consider the following equation and diagram in Figure 1:

$$L(x, x') = \delta(x, x') \left[E(x, x') + \int_S \rho_{x'}(x, x'') L(x', x'') \frac{\cos(\theta') \cos(\theta'')}{\|x' - x''\|^2} dx'' \right]$$

Explain what the terms $\delta(x, x')$, $E(x, x')$, S , $\rho_{x'}(x, x'')$, $\cos(\theta')$ and $\|x' - x''\|^2$ account for. What is the equation for?

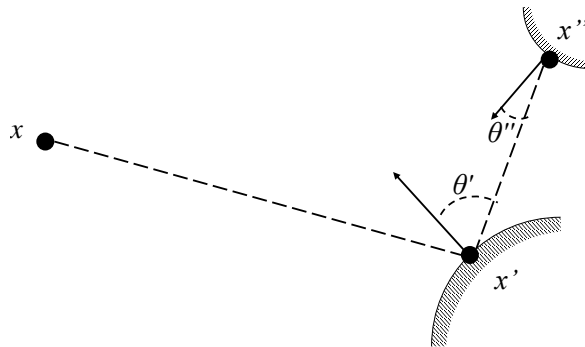


Figure 1: The illustration for the rendering equation.

5. (20%) (a) Explain what aliasing is in general. (5%) (b) What visual artifacts does aliasing could cause in rendering? (5%) (c) Suggest a method for reducing such artifacts. (10%)