

資訊網路與多媒體研究所

資格考試科目：資訊理論與編碼技巧

Information Theory and Coding Technique

2015/10/2

1. Prove that ( $H$  and  $D$  are the entropy and the relative entropy functions, and  $X, Y, Z$  are three different random variables, respectively.)

$$(1) H(X, Y | Z) = H(X | Z) + H(Y | X, Z).$$

$$(2) H(Y | X) \neq H(X | Y).$$

$$(3) H(X) - H(X | Y) = H(Y) - H(Y | X).$$

$$(4) D(p(x, y) \| q(x, y)) = D(p(x) \| q(x)) + D(p(y | x) \| q(y | x)).$$

2. A function  $\rho(x, y)$  is a metric if for all  $x, y$ ,

- $\rho(x, y) \geq 0$ .
- $\rho(x, y) = \rho(y, x)$ .
- $\rho(x, y) = 0$  if and only if  $x = y$ .
- $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$ .

- (1) Show that  $\rho(X, Y) = H(X | Y) + H(Y | X)$  satisfies the first, second, and fourth properties above. If we say that  $X = Y$  if there is one-to-one function mapping  $X$  to  $Y$ , then the third property is also satisfied, and  $\rho(X, Y)$  is a metric.

- (2) Verify that  $\rho(X, Y)$  can also be expressed as

$$\begin{aligned} \rho(X, Y) &= H(X) + H(Y) - 2I(X; Y) \\ &= H(X, Y) - I(X; Y) \\ &= 2H(X, Y) - H(X) - H(Y). \end{aligned}$$

3. Find the Ternary Huffman codes of the follow discrete sources

$$(a) \{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.15), (5, 0.15)\}$$

$$(b) \{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.1), (5, 0.1), (6, 0.1)\}$$

where  $(x, p)$  stands for the source symbol and its associated probability.

4. Which of the following codes are

$$C_1 = \{00, 01, 0\}$$

- (a) Uniquely decodable?

$$C_2 = \{00, 01, 100, 101, 11\}$$

- (b) Instantaneous decodable?

$$C_3 = \{0, 10, 110, 1110, \dots\}$$

$$C_4 = \{0, 00, 000, 0000\}$$

5. Please draw the block diagrams for the following two coding standards:
  - (a) JPEG Encoder and Decoder
  - (b) MPEG-1 Encoder and Decoder
  - (c) Explain briefly the function of each component in your diagrams of codecs.