Computer Graphics Ph.D. Qualifying Exam, March 2014

1. (20%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[I_{l_i} \left(k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s} \right) \min \left(1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

   Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

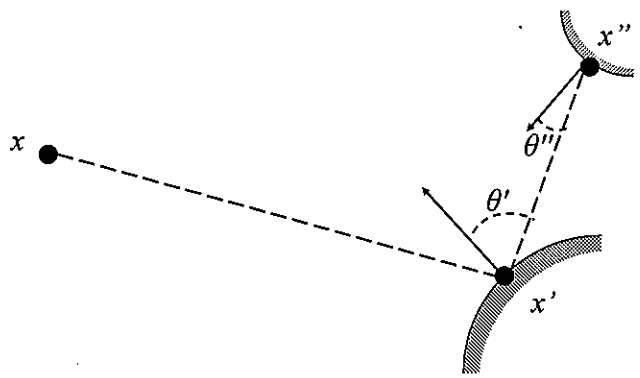
2. (20%) Given a parametric line $\mathrm{l}(t) = \mathbf{a} + t\mathbf{b}$ and an implicit sphere $s(\mathbf{x}) : \|\mathbf{x} - \mathbf{c}\| = r$, find t where the line is closest to the sphere.

3. (20%) $\mathbf{A}$ and $\mathbf{B}$ are 3×3 rotation matrices. Let $\mathbf{C}$ be a matrix created by a convex linear combination of them, $\mathbf{C} = \alpha\mathbf{A} + (1 - \alpha)\mathbf{B}$. Under what circumstances will $\mathbf{C}$ be a rotation matrix?

4. (20%) Aliasing refers to the artifact when the signal reconstructed from samples is different from the original signal. Describe why it happens and what artifacts in computer graphics are attributed to it. How to alleviate the problem?

5. (20%) Consider the following equation and diagram:

$$L(x, x') = \delta(x, x') \left[E(x, x') + \int_S \rho_{x'}(x, x'') L(x', x'') \frac{\cos(\theta') \cos(\theta'')}{\|x' - x''\|^2} dx'' \right]$$



   Explain what the terms $\delta(x, x')$, $E(x, x')$, S, $\rho_{x'}(x, x'')$, $\cos(\theta')$ and $\|x' - x''\|^2$ account for. What is the equation for?