

Computer Graphics Ph.D. Qualifying Exam, March 2013

1. (20%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[I_i \left(k_d (\mathbf{N} \cdot \mathbf{L}_i)_+ + k_s (\mathbf{V} \cdot \mathbf{R}_i)_+^{n_s} \right) \min \left(1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

2. (20%) The above equation does not account for shadows. Shadow mapping is a method for adding shadows for local shading. Describe how it works. What are its pros and cons?
3. (20%) (1) Given a ray whose origin is $\{0, 0, 0\}$ and direction is $\mathbf{d} = \{d_x, d_y, d_z\}$, express the coordinate of a point \mathbf{p} on the ray using its distance t to the ray origin. (2) Given a sphere with the center $\mathbf{c} = \{c_x, c_y, c_z\}$ and radius r , the intersection of the sphere and the above ray can be found by solving a quadratic equation of t , $At^2 + Bt + C = 0$. What are A , B and C ?
4. (20%) The following is Kajiya's rendering equation. Draw a diagram to explain the main terms in the equation and the meaning behind the formulation.

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{S^2} f(p, \omega_o, \omega_i) L_i(p, \omega_i) |\cos \theta_i| d\omega_i$$

5. (20%) Solving the above rendering equation requires calculating complex integrals, which can be estimated using Monte Carlo method. Assume that we want to estimate the integral, $I = \int_a^b f(x) dx$. Monte Carlo estimator draws N samples X_i uniformly from the interval $[a, b]$ and estimates I with the following formula,

$$F_N = \frac{b-a}{N} \sum_{i=1}^N f(X_i).$$

Show that the expected value of F_N equals I .