

資訊網路與多媒體研究所

資格考試科目：資訊理論與編碼技巧

Information Theory and Coding Technique

2013/9/26

1. (16%) A function $\rho(x, y)$ is a metric if for all x, y ,

- $\rho(x, y) \geq 0$.
- $\rho(x, y) = \rho(y, x)$.
- $\rho(x, y) = 0$ if and only if $x = y$.
- $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$.

(1) Show that $\rho(X, Y) = H(X|Y) + H(Y|X)$ satisfies the first, second, and fourth properties above. If we say that $X = Y$ if there is one-to-one function mapping X to Y , then the third property is also satisfied, and $\rho(X, Y)$ is a metric.

(2) Verify that $\rho(X, Y)$ can also be expressed as

$$\begin{aligned}\rho(X, Y) &= H(X) + H(Y) - 2I(X; Y) \\ &= H(X, Y) - I(X; Y) \\ &= 2H(X, Y) - H(X) - H(Y).\end{aligned}$$

2. (6%) Let a random variable $X = \begin{cases} 0, & p(0) = P \\ 1, & p(1) = 1 - P \end{cases}$,

where p is a probability function. Please prove the following:

- (1) $H(X) = 1$ bits when $P = 1/2$.
- (2) $H(X)$ is a concave function of P .
- (3) $H(X) = 0$ if $P = 0$ or 1 .
- (4) The max $H(X)$ occurs when $P = 1/2$.

3. (8%) Find the Ternary Huffman codes of the follow discrete sources

- (a) $\{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.15), (5, 0.15)\}$
- (b) $\{(1, 0.25), (2, 0.25), (3, 0.2), (4, 0.1), (5, 0.1), (6, 0.1)\}$

where (x, p) stands for the source symbol and its associated probability.

4. (15%) Show that the expected length under $p(x)$ of the code assignment

$$l(x) = \left\lceil \log \frac{1}{q(x)} \right\rceil \text{ satisfies}$$

$$H(p) + D(p // q) \leq Epl(x) < H(p) + D(p // q) + 1$$

where $\lceil x \rceil$ is the smallest integer larger than or equal to X .

5. (15%) Prove the convexity and/or concavity of the following functions:
 - (a) $H(p)$ is a concave function of P .
 - (b) $D(p//q)$ is convex in the pair (p,q)
 - (c) $I(X;Y)$ is a concave function of $p(x)$ for fixed $p(y|x)$
 - (d) $I(X;Y)$ is a convex function of $p(y|x)$ for fixed $p(x)$.

6. (15%) Please draw the block diagrams for the following two coding standards:
 - (a) JPEG Encoder and Decoder
 - (b) MPEG-1 Encoder and Decoder
 - (c) Explain briefly the function of each component in your diagrams of codecs.

7. (5%) Please briefly explain why and under what conditions DPCM will provide compression gain?

8. (5%) Please briefly explain why and under what conditions Transform coding will provide compression gain?

9. (5%) What is 3-step fast Search Algorithm?
Please describe the pros and cons of 3-step fast Search Algorithm?

10. (20%) Try your best to derive all the relationships between DCT and discrete Fourier Transform (DFT), where an N -point DFT is defined as

$$F(X) \triangleq \bar{X} = \sum_{n=0}^{N-1} x(n)W_N^{nk}$$
 and $W_N^k = \exp\left(-j\frac{2\pi}{N}k\right)$