

資訊網路與多媒體研究所

博士班資格考試科目：資訊理論與編碼技巧

2011/3/10

1. (15%) The World Series is a seven-game series that terminates as soon as either team wins four games. Let X be the random variable that represents the outcome of a World Series between teams A and B; possible values of X are AAAA, BABABAB, and BBBAAAA. Let Y be the number of games played, which ranges from 4 to 7. Assuming that A and B are equally matched and that the games are independent, calculate $H(X)$, $H(Y)$, $H(Y|X)$, and $H(X|Y)$.
2. (15%) Find the Ternary Huffman codes of the follow discrete sources
 - (a) $\{(1,0.25), (2,0.25), (3,0.2), (4,0.15), (5,0.15)\}$
 - (b) $\{(1,0.25), (2,0.25), (3,0.2), (4,0.1), (5,0.1), (6,0.1)\}$where (x, p) stands for the source symbol and its associated probability.
3. (15%) Prove that if f is a convex function and X is a random variable
$$E f(x) \geq f(EX)$$
where E stands for expectation.
4. (15%) Please find the differential entropy of a continuous Gaussian random variable X , where $X \sim \varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$
5. (15%) Prove that (H and D are the entropy and the relative entropy functions, and X, Y, Z are three different random variables, respectively.)
 - (1) $H(X, Y | Z) = H(X | Z) + H(Y | X, Z)$.
 - (2) $H(Y | X) \neq H(X | Y)$.
 - (3) $H(X) - H(X | Y) = H(Y) - H(Y | X)$.
 - (4) $D(p(x, y) \| q(x, y)) = D(p(x) \| q(x)) + D(p(y | x) \| q(y | x))$.
6. (10%) Please draw the basic structure of a motion-compensated video coding scheme. (including both Encoder and Decoder)
7. (15%) Let $X_1 \rightarrow X_2 \rightarrow \dots \rightarrow X_n$ form a Markov chain in this order:
That is, let $p(x_1, x_2, \dots, x_n) = p(x_1) p(x_2|x_1) \dots p(x_n|x_{n-1})$.
Reduce $I(X_1; X_2, X_3, \dots, X_n)$ to its simplest form.