

資訊網路與多媒體研究所

資格考科目：資訊理論與編碼技巧

2010/9/30

1. (15%) Let $\bar{X} = \{0,1\}$ and consider two distributions p and q on \bar{X} .
Let $p(0)=1-r$ and $q(a)=s$. Then
 - (a) $D(p//q)=?$
 - (b) $D(q//p)=?$
 - (c) Under what condition $D(p//q)=D(q//p)$?

2. (15%) Prove the following Theorems.
 - (a) Chain rule for relative entropy
$$D(p(x,y)//q(x,y))=D(p(x)//q(x))+D(p(y|x)//q(y|x))$$
 - (b) Jensen's Inequality
If f is a convex function and X is a random variable, then $Ef(x) \geq f(EX)$.
 - (c) $H(X|Y) \leq H(X) \leq H(X,Y)$

3. (10%) For a stationary Stochastic process X_1, X_2, \dots, X_n ,
Show that
 - (a) $\frac{H(X_1, X_2, \dots, X_n)}{n} \leq \frac{H(X_1, X_2, \dots, X_{n-1})}{n-1}$
 - (b) $\frac{H(X_1, X_2, \dots, X_n)}{n} \geq H(X_n | X_{n-1}, \dots, X_1)$

4. (15%) Construct the optimal Huffman Codes for the following
Three Sources:
 - (a) Binary Huffman Code for $\left\{ \begin{matrix} X_1 & X_2 & X_3 & X_4 & X_5 \\ 0.25 & 0.25 & 0.2 & 0.15 & 0.15 \end{matrix} \right\}$
 - (b) Ternary Huffman Code for $\left\{ \begin{matrix} Y_1 & Y_2 & Y_3 & Y_4 & Y_5 & Y_6 \\ 0.25 & 0.25 & 0.2 & 0.1 & 0.1 & 0.1 \end{matrix} \right\}$
 - (c) Binary Huffman Code for $\left\{ \begin{matrix} Z_1 & Z_2 & Z_3 & Z_4 & Z_5 \\ 0.4 & 0.2 & 0.2 & 0.1 & 0.1 \end{matrix} \right\}$

such that the effect of Noise in Huffman coding probabilities is minimal.

5. (5%) Please draw the basic structure of a motion-compensated video coding scheme. (including both Encoder and Decoder)

