1. (15%) A and B are $3 \times 3$ rotation matrices. Let $C$ be a matrix created by a convex linear combination of them ($C = \alpha A + (1 - \alpha)B$). Under what circumstances will $C$ be a rotation matrix?

2. (15%) Consider a unit square. Assume that the texture coordinates are $(u_{00}, v_{00}), (u_{10}, v_{10}), (u_{01}, v_{01}), (u_{11}, v_{11})$ for its four vertices $(0, 0), (1, 0), (0, 1), (1, 1)$. Given a point $(x, y)$ within the square, what is its texture coordinate using bilinear interpolation?

3. (10%) A mipmap is an data structure for texture map anti-aliasing. Explain how it works and estimate its memory consumption compared to the initial texture image alone.

4. (20%) (a) The Phong illumination model can be summarized by the following equation:

$$I = k_e + k_a I_a + \sum_i \left[ I_i \left( k_d (N \cdot L_i) + k_s (V \cdot R_i) \right) \min \left( 1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right) \right]$$

Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

5. (15%) Given a triangle $T$ with vertices $V_1, V_2, V_3$ and a ray $R$ with origin $O$ and direction $d$, (a) what is the plane equation where $T$ lies? (5%), (b) what is the intersection of $R$ and that plane? (5%) and (c) how to check whether $R$ and $T$ intersect? (5%)

6. (25%) (a) Describe the rendering equation proposed by Kajiya in his classic SIGGRAPH 1986 paper. (10%) (b) Explain how to derive Whitted’s model from the rendering equation. (10%) (c) Could you suggest a way to find the solution to the rendering equation without making assumptions. You can ignore the efficiency issue. (5%)