Computer Graphics Ph.D. Qualifying Exam, March 2011

1. (20%) Write down the matrix \( M \) that takes the orthonormal 3D vectors \( \mathbf{u} = (x_u, y_u, z_u) \), \( \mathbf{v} = (x_v, y_v, z_v) \) and \( \mathbf{w} = (x_w, y_w, z_w) \) to orthonormal 3D vectors \( \mathbf{a} = (x_a, y_a, z_a) \), \( \mathbf{b} = (x_b, y_b, z_b) \) and \( \mathbf{c} = (x_c, y_c, z_c) \), so that \( M\mathbf{u} = \mathbf{a} \), \( M\mathbf{v} = \mathbf{b} \), and \( M\mathbf{w} = \mathbf{c} \). You do not need to write down the explicit matrix if it involves matrix inverse or transpose.

2. (20%) (a) The Phong illumination model can be summarized by the following equation:

\[
I = k_c + k_a I_a + \sum_i \left[ I_i d_c (N \cdot L_i)_+ + k_d (V \cdot R_i)_+ \right] \min \left( 1, \frac{1}{a_0 + a_1 d_i + a_2 d_i^2} \right)
\]

Draw a diagram to explain the main variables in the above formulation. What effects do the terms of the above formulation intend to model? (b) Describe how to shade a triangle using flat shading, Gouraud shading and Phong shading. Discuss their visual differences.

3. (20%) What is the average value of the function \( xyz \) in the unit cube \((x, y, z) \in [0, 1]^3\)?

4. (20%) Given a sphere with the origin as its center and \( r \) as its radius, describe algebraically how to determine its intersections with a ray whose origin is \( o = \{o_x, o_y, o_z\} \) and direction is \( d = \{d_x, d_y, d_z\} \).

5. (20%) Assume a very simple scene, the inside of a uniformly emitting Lambertian sphere with emittance \( E \) and reflectance \( \rho \). What is the radiance for each point inside the sphere when the scene reaches equilibrium.